Name________________ Class________________ Date________________

4-1
COMMON CORE
CC.9-12.A.CED.2*, CC.9-12.F.IF.2, CC.9-12.F.IF.4*, CC.9-12.F.IF.7d(+)*, CC.9-12.F.BF.1*, CC.9-12.F.BF.3

Graphing $f(x) = \frac{a}{x}$

(handwritten: fraction)

Essential question: *What is the effect of changing the value of a on the graph of $f(x) = \frac{a}{x}$?*

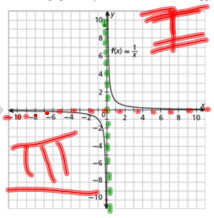
1 ENGAGE Understanding the Parent Function $f(x) = \frac{1}{x}$

(handwritten: $\frac{1}{0}$ Undefined)

The function $f(x) = \frac{1}{x}$ is the parent function of all functions of the form $g(x) = \frac{a}{x}$. The graph of $f(x) = \frac{1}{x}$ consists of two separate curves, one in Quadrant III and one in Quadrant I, called *branches*. As you can see from the tables and graph below, the ends of the branches approach the axes, which are called the graph's *asymptotes*.

$x < 0$	
x	$f(x)$
−5	$-\frac{1}{5}$
−2	$-\frac{1}{2}$
−1	−1
$-\frac{1}{2}$	−2
$-\frac{1}{5}$	−5

$x > 0$	
x	$f(x)$
$\frac{1}{5}$	5
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
5	$\frac{1}{5}$



(handwritten: Asymptotes — V. Asymptotes $x = 0$; H. Asymptotes $y = 0$)

REFLECT

1a. What are the domain and range of the function?

(handwritten: $D: x \in \mathbb{R} : x \neq 0$ $R: y \in \mathbb{R} : y \neq 0$)

1b. Is the function increasing or decreasing for $x < 0$? Is it increasing or decreasing for $x > 0$?

____

1c. If a is a nonzero number, both a and $\frac{1}{a}$ have the same sign. What does this fact tell you about the graph of the function?

____

1d. If a is a nonzero number, then both $\left(a, \frac{1}{a}\right)$ and $\left(\frac{1}{a}, a\right)$ are points on the graph of the function. What does this fact tell you about the symmetry of the graph?

____

1e. The function's *end behavior* is determined by what happens to the value of $f(x)$ as the value of x increases or decreases without bound. The notation $x \to +\infty$, which is read "x approaches positive infinity," means that x is increasing without bound, while the notation $x \to -\infty$, which is read "x approaches negative infinity," means that x is decreasing without bound. Complete each table and then describe the function's end behavior.

x increases without bound.	
x	$f(x) = \frac{1}{x}$
100	
1000	
10,000	
100,000	

x decreases without bound.	
x	$f(x) = \frac{1}{x}$
−100	
−1000	
−10,000	
−100,000	

Feb 4-3:33 PM

As $x \to +\infty$, $f(x) \to$ ________. As $x \to -\infty$, $f(x) \to$ ________.

1f. The break in the function's graph at $x = 0$ is called an *infinite discontinuity*. To see why this is so, complete each table and then describe the function's behavior. The notation $x \to 0^+$ means that x approaches 0 from the right, while the notation $x \to 0^-$ means that x approaches 0 from the left.

x approaches 0 from the right.	
x	$f(x) = \frac{1}{x}$
0.01	
0.001	
0.0001	
0.00001	

As $x \to 0^+$, $f(x) \to$ ________.

x approaches 0 from the left.	
x	$f(x) = \frac{1}{x}$
−0.01	
−0.001	
−0.0001	
−0.00001	

As $x \to 0^-$, $f(x) \to$ ________.

2 EXAMPLE Graphing $g(x) = \frac{a}{x}$ when $a > 0$

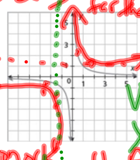
Graph each function. (The parent function is shown in gray.)

A $g(x) = \frac{2}{x}$

(handwritten: $a = 2$ V. Stretch BAFO 2 → Branches are farther from the x-axis)

$x < 0$	
x	$g(x) = \frac{2}{x}$
−4	
−2	−1
−1	−2
$-\frac{1}{2}$	

$x > 0$	
x	$g(x) = \frac{2}{x}$
$\frac{1}{2}$	
1	2
2	1
4	



(handwritten: V. Asym. $x = 0$; H. Asym. $y = 0$)

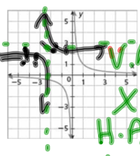
(handwritten: V. Asymp set the denominator = 0; H. Asympt after the fraction or K; $x = 0$ ... 4)

B $g(x) = \frac{0.4}{x}$

(handwritten: V. shrink BAFO 0.4)

$x < 0$	
x	$g(x) = \frac{0.4}{x}$
−2	−0.2
−1	−0.4
−0.4	
−0.2	
−0.1	

$x > 0$	
x	$g(x) = \frac{0.4}{x}$
0.1	
0.2	
0.4	
1	0.4
2	0.2



(handwritten: V. Asym. $x = 0$; H. Asym. $y = 0$)

REFLECT

2a. You can obtain the graph of $g(x) = \frac{a}{x}$ from the graph of $f(x) = \frac{1}{x}$ by vertically stretching or shrinking it. Use this fact to complete the table.

Value of a in $g(x) = \frac{a}{x}$	Vertical stretch or shrink of the graph of f
$a > 1$	
$0 < a < 1$	

3 EXAMPLE Graphing $g(x) = \frac{a}{x}$ when $a < 0$

Graph each function. (The parent function is shown in gray.)

A $g(x) = -\frac{2}{x}$

Feb 4-3:33 PM

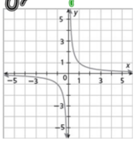
$x < 0$	
x	$g(x) = -\frac{2}{x}$
−4	
−2	1
−1	2
$-\frac{1}{2}$	

$x > 0$	
x	$g(x) = -\frac{2}{x}$
$\frac{1}{2}$	
1	−2
2	−1
4	

B $g(x) = -\frac{0.4}{x}$

$x < 0$	
x	$g(x) = -\frac{0.4}{x}$
−2	
−1	
−0.4	
−0.2	
−0.1	

$x > 0$	
x	$g(x) = -\frac{0.4}{x}$
0.1	
0.2	
0.4	
1	
2	



REFLECT

3a. Use the table below to summarize your comparisons of the graph of $g(x) = \frac{a}{x}$ with the graph of $f(x) = \frac{1}{x}$ for the given values of a.

Value of a in $g(x) = \frac{a}{x}$	Vertical stretch or shrink of the graph of f?	Also a reflection across the x-axis?
$a > 1$	V. stretch	No
$0 < a < 1$	V. shrink	No
$-1 < a < 0$	V. shrink	yes
$a < -1$	V. stretch	yes

Inverse Variation When the relationship between two real-world quantities x and y has the form $y = \frac{a}{x}$ for some nonzero constant a, the relationship is called *inverse variation* and y is said to *vary inversely* as x.

4 EXAMPLE Writing and Graphing an Equation for Inverse Variation

Mrs. Jacobs drives 30 miles to her job in the city. Her commuting time depends on her average speed, which varies from day to day as a result of and traffic conditions. Write and graph an equation that gives her commuting time as a function of her average speed.

A Use the formula $d = rt$ where d is distance, r is rate (average speed), and t is time to write t as a function of r given that $d = 30$.

$rt =$ ___ The product of rate and time gives distance.

$t = \frac{\quad}{r}$ Solve for t.

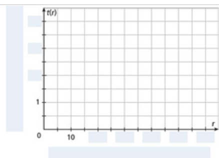
B Use the table to help you graph the function $t(r)$.

$D = rt$

$\frac{30}{r} = t$

Feb 4-3:34 PM

r	$t(r)$
10	
15	
30	
60	



REFLECT

4a. Why does the graph consist only of the branch in Quadrant I?

4b. Do equal changes in average speed result in equal changes in commuting time? Give an example to support your answer.

PRACTICE

For each function, plot the points at which $x = \pm 1$, then draw the complete graph.

1. $f(x) = \frac{0.3}{x}$ **2.** $f(x) = -\frac{4}{x}$

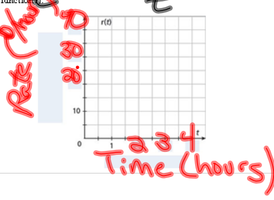
3. Shaun is paid $20 each week to mow a lawn. The time he spends mowing varies from week to week based on factors such as how much the grass has grown and how wet the grass is. His effective hourly pay rate is therefore a function of the time he spends mowing.

a. Use the formula $p = rt$ where p is total pay, r is hourly pay rate, and t is time to write r as a function of t given that $p = 20$. Describe the relationship between r and t.

$p = r \cdot t$ $\quad \frac{20}{t} = \frac{r \cdot t}{t}$ $\quad \frac{20}{t} = r$

b. Use the table below to help you graph the function $r(t)$.

t	$r(t)$
0.5	40
1	20
2	10
2.5	8



Feb 4-3:34 PM

May 1-2:24 PM

Name____________________ Class____________ Date__________

4-1

COMMON CORE
CC.9-12.A.CED.2*,
CC.9-12.F.IF.2,
CC.9-12.F.IF.4*,
CC.9-12.F.IF.7d(+)*,
CC.9-12.F.BF.1*,
CC.9-12.F.BF.3

Graphing $f(x) = \frac{a}{x}$.

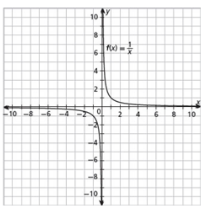
Essential question: *What is the effect of changing the value of a on the graph of $f(x) = \frac{a}{x}$?*

1 ENGAGE Understanding the Parent Function $f(x) = \frac{1}{x}$

The function $f(x) = \frac{1}{x}$ is the parent function of all functions of the form $g(x) = \frac{a}{x}$. The graph of $f(x) = \frac{1}{x}$ consists of two separate curves, one in Quadrant III and one in Quadrant I, called *branches*. As you can see from the tables and graph below, the ends of the branches approach the axes, which are called the graph's *asymptotes*.

$x < 0$	
x	$f(x)$
−5	$-\frac{1}{5}$
−2	$-\frac{1}{2}$
−1	−1
$-\frac{1}{2}$	−2
$-\frac{1}{5}$	−5

$x > 0$	
x	$f(x)$
$\frac{1}{5}$	5
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
5	$\frac{1}{5}$



REFLECT

1a. What are the domain and range of the function?

The domain is $\{x \mid x \neq 0\}$; the range is $\{y \mid y \neq 0\}$.

1b. Is the function increasing or decreasing for $x < 0$? Is it increasing or decreasing for $x > 0$?

Decreasing; decreasing

1c. If a is a nonzero number, both a and $\frac{1}{a}$ have the same sign. What does this fact tell you about the graph of the function?

The graph must lie only in Quadrant I and Quadrant III where x and y have the same sign.

1d. If a is a nonzero number, then both $\left(a, \frac{1}{a}\right)$ and $\left(\frac{1}{a}, a\right)$ are points on the graph of the function. What does this fact tell you about the symmetry of the graph?

The graph is symmetric about the line $y = x$.

1e. The function's *end behavior* is determined by what happens to the value of $f(x)$ as the value of x increases or decreases without bound. The notation $x \to +\infty$, which is read "x approaches positive infinity," means that x is increasing without bound, while the notation $x \to -\infty$, which is read "x approaches negative infinity," means that x is decreasing without bound. Complete each table and then describe the function's end behavior.

x increases without bound.	
x	$f(x) = \frac{1}{x}$
100	0.01

x decreases without bound.	
x	$f(x) = \frac{1}{x}$
−100	− 0.01

Feb 4-3:36 PM

1000	0.001
10,000	0.0001
100,000	0.00001

−1000	−0.001
−10,000	−0.0001
−100,000	−0.00001

As $x \to +\infty, f(x) \to$ 0.

As $x \to -\infty, f(x) \to$ 0.

1f. The break in the function's graph at = 0 is called an *infinite discontinuity*. To see why this is so, complete each table and then describe the function's behavior
The notation $\to 0^+$ means that x approaches 0 from the right, while the notation $\to 0^-$ means that x approaches 0 from the left.

x approaches 0 from the right.	
x	$f(x) = \frac{1}{x}$
0.01	100
0.001	1000
0.0001	10,000
0.00001	100,000

x approaches 0 from the left.	
x	$f(x) = \frac{1}{x}$
−0.01	−100
−0.001	−1000
−0.0001	−10,000
−0.00001	−100,000

As $x \to 0^+, f(x) \to$ $+\infty$

As $x \to 0^-, f(x) \to$ $-\infty$

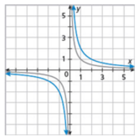
2 EXAMPLE Graphing $g(x) = \frac{a}{x}$ when a > 0

Graph each function. (The parent function is shown in gray.)

A $g(x) = \frac{2}{x}$

$x < 0$	
x	$g(x) = \frac{2}{x}$
−4	$-\frac{1}{2}$
−2	−1
−1	−2
$-\frac{1}{2}$	−4

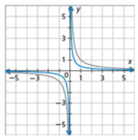
$x > 0$	
x	$g(x) = -\frac{2}{x}$
$\frac{1}{2}$	4
1	2
2	1
4	$\frac{1}{2}$



B $g(x) = \frac{0.4}{x}$

$x < 0$	
x	$g(x)\ \frac{0.4}{x}$
−2	−0.2
−1	−0.4
−0.4	−1
−0.2	−2
−0.1	−4

$x > 0$	
x	$g(x)\ \frac{0.4}{x}$
0.1	4
0.2	2
0.4	1
1	0.4
2	0.2



REFLECT

2a. You can obtain the graph of $g(x) = \frac{a}{x}$ from the graph of $f(x) = \frac{1}{x}$ by vertically stretching or shrinking it. Use this fact to complete the table.

Feb 4-3:36 PM

Value of a in $g(x) = \frac{a}{x}$	Vertical stretch or shrink of the graph of f?
$a > 1$	Vertical stretch
$0 < a < 1$	Vertical shrink

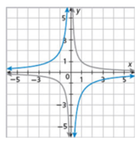
3 EXAMPLE Graphing $g(x) = \frac{a}{x}$ when $a < 0$

Graph each function. (The parent function is shown in gray.)

A $g(x) = -\frac{2}{x}$

$x < 0$	
x	$g(x) = -\frac{2}{x}$
−4	$\frac{1}{2}$
−2	1
−1	2
$-\frac{1}{2}$	4

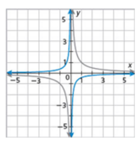
$x > 0$	
x	$g(x) = -\frac{2}{x}$
$\frac{1}{2}$	−4
1	−2
2	−1
4	$-\frac{1}{2}$



B $g(x) = -\frac{0.4}{x}$

$x < 0$	
x	$g(x) = -\frac{0.4}{x}$
−2	0.2
−1	0.4
−0.4	1
−0.2	2
−0.1	4

$x > 0$	
x	$g(x) = -\frac{0.4}{x}$
0.1	−4
0.2	−2
0.4	−1
1	−0.4
2	−0.2



REFLECT

3a. Use the table below to summarize your comparisons of the graph of $g(x) = \frac{a}{x}$ with the graph of $f(x) = \frac{1}{x}$ for the given values of a.

Value of a in $g(x) = \frac{a}{x}$	Vertical stretch or shrink of the graph of f?	Also a reflection across the x-axis?
$a > 1$	Vertical stretch	No
$0 < a < 1$	Vertical shrink	No
$-1 < a < 0$	Vertical shrink	Yes
$a < -1$	Vertical stretch	Yes

Inverse Variation When the relationship between two real-world quantities and y has the form $y = \frac{a}{x}$ for some nonzero constant the relationship is called *inverse variation* and y is said to *vary inversely* as x.

4 EXAMPLE Writing and Graphing an Equation for Inverse Variation

Mrs. Jacobs drives 30 miles to her job in the city. Her commuting time depends on her average speed, which varies from day to day as a result of and traffic conditions. Write and graph an equation that gives her commuting time as a function of her average speed.

A Use the formula $d = rt$ where d is distance, r is rate (average speed), and t is time to write t as a function of r given that $d = 30$.

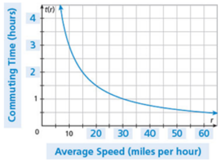
$rt =$ 30 The product of rate and time gives distance.

$t = \frac{30}{r}$ Solve for t.

Feb 4-3:36 PM

B Use the table to help you graph the function $t(r)$.

r	$t(r)$
10	3
15	2
30	1
60	0.5



REFLECT

4a. Why does the graph consist only of the branch in Quadrant I?

Rate and time have only positive values in this problem.

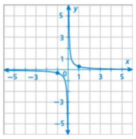
4b. Do equal changes in average speed result in equal changes in commuting time? Give an example to support your answer.

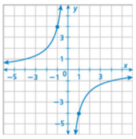
No; going from 10 mi/h to 20 mi/h decreases the commuting time by 1.5 h, but going from 20 mi/h to 30 mi/h decreases the commuting time by only 0.5 h.

PRACTICE

For each function, plot the points at which ±1, then draw the complete graph.

1. $f(x) = \frac{0.3}{x}$ **2.** $f(x) = -\frac{4}{x}$





3. Shaun is paid $20 each week to mow a lawn. The time he spends mowing varies from week to week based on factors such as how much the grass has grown and how wet the grass is. His effective hourly pay rate is therefore a function of the time he spends mowing.

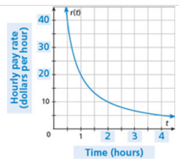
a. Use the formula $p = rt$ where p is total pay, r is hourly pay rate, and t is time to write r as a function of t given that $p = 20$. Describe the relationship between r and t.

$r = \frac{20}{t}$; r varies inversely as

b. Use the table below to help you graph the function $r(t)$.

Feb 4-3:36 PM

t	$r(t)$
0.5	40
1	20
2	10
2.5	8



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