Name ______________ Class ______ Date ______

3-1

Investigating the Graph of $f(x) = x^n$

COMMON CORE

CC9-12.F.IF.7*, CC9-12.F.IF.7c*, CC9-12.F.BF.3

Essential question: *How does the value of n affect the behavior of the function $f(x) = x^n$?*

1 EXPLORE Graphing $f(x) = x^n$ When n is Even

Follow these steps to investigate the graphs of $f(x) = x^2$, $f(x) = x^4$, and $f(x) = x^6$.

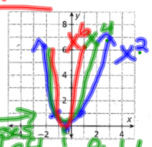
A Set the viewing window of your graphing calculator as shown.

```
WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-1
Ymax=9
Yscl=1
Xres=1
```

B Enter the functions $f(x) = x^2$, $f(x) = x^4$, and $f(x) = x^6$ in the equation editor as shown.

```
Plot1 Plot2 Plot3
\Y1=X^2
\Y2=X^4
\Y3=X^6
\Y4=
\Y5=
\Y6=
\Y7=
```

C Graph the functions on the coordinate plane at right by sketching what you see on your calculator.



D Use your graphs to identify the zero(s) of the functions. crosses the x-axis

x=0 (0,0)

E Identify the minimum value of each function.

y=0

F Describe any symmetry of the graphs.

y-axis

REFLECT

1a. What do all of the functions and their graphs have in common?

x=0 (zero), minimum y=0, x-int, y-axis symmetry

End Behavior Even

1b. For these functions, what happens to the values of $f(x)$ as x increases without bound? How is this displayed in the graph?

① x → +∞ f(x) → +∞ ② graph x-axis left right neg pos x-decreases Even

1c. For these functions, what happens to the values of $f(x)$ as x decreases without bound? How is this displayed in the graph?

x → −∞ f(x) → +∞

2 EXPLORE Graphing $f(x) = x^n$ When n is Odd

Follow these steps to investigate the graphs of $f(x) = x$, $f(x) = x^3$, and $f(x) = x^5$.

A Set the viewing window of your graphing calculator as shown.

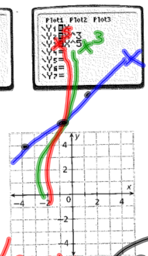
```
WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Xres=1
```

B Enter the functions $f(x) = x$, $f(x) = x^3$, and $f(x) = x^5$ in the equation editor as shown.

```
Plot1 Plot2 Plot3
\Y1=X
\Y2=X^3
\Y3=X^5
\Y4=
\Y5=
\Y6=
\Y7=
```

C Graph the functions on the coordinate plane at right by sketching what you see on your calculator.

x=0 (0,0)



D Use your graphs to identify the zero(s) of the functions.

None

E Identify any maximum values or minimum values of each function.

origin symmetry

F Describe any symmetry of the graphs.

① x=0 zeros, No min/max, symmetry at the origin Even

REFLECT

2a. What do all of the functions and their graphs have in common?

Odd (0,0) (1,1) (−1,−1) : (0,0) (1,1) (−1,−1)

2b. What points lie on all of the graphs?

2c. For these functions, what happens to the values of $f(x)$ as x increases without bound? How is this displayed in the graph?

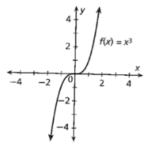
End Behavior x → +∞ f(x) → +∞

x → −∞ f(x) → −∞

2d. For these functions, what happens to the values of $f(x)$ as x decreases without bound? How is this displayed in the graph?

3 ENGAGE Describing Characteristics of Functions

The **end behavior** of a function is a description of the values of the function as x increases without bound or decreases without bound.

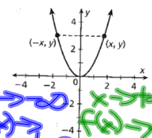
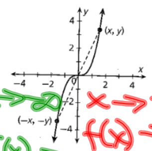


For example, for $f(x) = x^3$, the values of $f(x)$ increase without bound as x increases without bound. You can say that $f(x)$ approaches positive infinity as x approaches positive infinity. This may be abbreviated as "$f(x) \to +\infty$ as $x \to +\infty$."

Also, the values of $f(x)$ decrease without bound as x decreases without bound. You can say that $f(x)$ approaches negative infinity as x approaches negative infinity. This may be abbreviated as "$f(x) \to -\infty$ as $x \to -\infty$."

A function is an **even function** if $f(-x) = f(x)$ for all values of x. This means that if the point (x, y) is on the graph, then the point $(-x, y)$ is also on the graph, so the graph is symmetric with respect to the y-axis.

A function is an **odd function** if $f(-x) = -f(x)$ for all values of x. This means that if the point (x, y) is on the graph, then the point $(-x, -y)$ is also on the graph, so the graph has 180° rotational symmetry about the origin.

The Graph of an Even Function	The Graph of an Odd Function
	

$x \to -\infty$ $f(x) \to +\infty$ $x \to +\infty$ $f(x) \to +\infty$ $x \to -\infty$ $f(x) \to -\infty$ $x \to +\infty$ $f(x) \to \infty$

REFLECT

3a. For the function $g(x)$, you are told that $g(1000) = 5{,}000{,}000$. Is it possible to make any conclusions about the end behavior of $g(x)$? Explain.

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3b. What can you say about the end behavior of $f(x) = x^n$ when n is even?

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3c. What can you say about the end behavior of $f(x) = x^n$ when n is odd?

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3d. Explain why any function of the form $f(x) = x^n$ is an even function if n is even.

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3e. Explain why any function of the form $f(x) = x^n$ is an odd function if n is odd.

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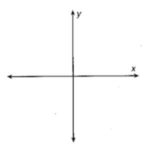
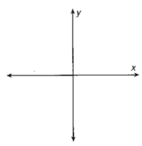
3f. Complete the table.

Characteristics of $f(x) = x^n$		
	n is even	n is odd
Sketch of graph of $f(x) = x^n$		
End behavior	As $x \to +\infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____	As $x \to +\infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____
Zeros	$x =$ _____	$x =$ _____
Maximum or minimum values	Maximum: _____ Minimum: _____	Maximum: _____ Minimum: _____
Symmetry		
Even or odd function		